29-Jan

Wednesday, January 29, 2025 1:49 PM

Lecture 2

§1.1 cont'd

ex. LS with infinitely many solutions

$$\begin{array}{l|l|l} \mathrm{I} & x + 2y + 3z = 0 & \\ \mathrm{II} & 4x + 5y + 6z = 3 & + (-4)\mathrm{I} \\ \mathrm{III} & 7x + 8y + 9z = 6 & + (-7)\mathrm{I} \end{array}$$

recall: sys. of eq'ns have:
- single solution
- infinitely many sol'ns
- no solutions

$$\begin{array}{r} -4x - 8y - 12z = 0 \\ 4x + 5y + 6z = 3 \\ \hline -3y - 6z = 3 \end{array}$$

$$\downarrow$$

$$\begin{array}{l|l|} \mathrm{I} & x + 2y + 3z = 0 \\ & -3y - 6z = 3 \\ & -6y - 12z = 6 \end{array} \leftarrow *(-2)$$

$$\begin{array}{r} -7x - 14y - 21z = 0 \\ 7x + 8y + 9z = 6 \\ \hline -6y - 12z = 6 \end{array}$$

$$\downarrow$$

$$\begin{array}{|l|} x + 2y + 3z = 0 \\ 6y + 12z = -6 \\ -6y - 12z = 6 \end{array}$$

$$\downarrow$$

$$\begin{array}{|l|} x + 2y + 3z = 0 \\ y + 2z = -1 \\ 0 = 0 \end{array} \leftarrow *(-2) \rightarrow \quad \begin{array}{r} x + 2y + 3z = 0 \\ -2y - 4z = 2 \\ \hline x \quad\quad - z = 2 \end{array}$$

$$\downarrow$$

goal:

$$\begin{array}{|l|} x \quad\quad = \sim \\ \quad y \quad = \sim \\ \quad\quad z = \sim \end{array}$$

$$\begin{array}{|l|} x \quad\quad - z = 2 \\ y + 2z = -1 \\ 0 = 0 \end{array} \xrightarrow[\text{ITO } z]{\text{write } x, y} \begin{array}{|l|} x = z + 2 \\ y = -2z - 1 \end{array}$$

select z = arbitrary t

then $x = t + 2$
$y = -2t - 1$
$z = t$

$$\longrightarrow \boxed{(\underset{x}{t+2}, \underset{y}{-2t-1}, \underset{z}{t})}$$

ex. solve LS w no solutions

$$\left|\begin{aligned} x+2y+3z&=0\\ 4x+5y+6z&=3\\ 7x+8y+9z&=0\end{aligned}\right|$$

same as previous example except of this value

$$\vdots$$

$$\left|\begin{aligned} x\qquad -z&=2\\ y+2z&=-1\\ 0&=-6\end{aligned}\right|$$

false statement, which results in an inconsistent system
∴ system yields <u>no</u> <u>solutions</u>

§1.2 Matrices, Vectors, and Gauss-Jordan Elimination

streamline a linear system by assuming the variables
i.e., write only the coefficients and constants

ex. $$\left|\begin{aligned} 3x+21y-3z&=0\\ -6x-2y-z&=62\\ 2x-3y+8z&=32\end{aligned}\right| \rightarrow \begin{array}{cccc} x & y & z & c\\ \end{array}\left[\begin{array}{ccc|c} 3 & 21 & -3 & 0\\ -6 & -2 & -1 & 62\\ 2 & -3 & 8 & 32\end{array}\right]$$

surround with brackets in this format

3 x 4 matrix

\# of rows i \# of columns j

dashed line is sometimes used to denote that last column contains constants

generalize each "cell" : a_{ij}

$$\underset{3\times4}{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14}\\ a_{21} & a_{22} & a_{23} & a_{24}\\ a_{31} & a_{32} & a_{33} & a_{34}\end{bmatrix}$$

<u>Square</u> Matrix := when the dimensions of a matrix is $n\times n$

ex $B=\begin{bmatrix}1&2\\3&4\end{bmatrix}$ $C=\begin{bmatrix}2&0&0\\0&3&0\\0&0&0\end{bmatrix}$ $D=\begin{bmatrix}2&3\\0&4\end{bmatrix}$ $E=\begin{bmatrix}5&0&0\\4&0&0\\3&2&1\end{bmatrix}$

2x2   3x3   2x2   3x3

diagonal

diagonal of a square matrix : $a_{11}, a_{22}, a_{33}, \dots, a_{nn}$

<u>Diagonal</u> <u>Matrix</u> := matrix when all entries <u>above</u> and <u>below</u> diagonal are zero

ex. matrix C is a diagonal matrix

<u>Lower Triangular</u> Matrix – all entries <u>above</u> diagonal are zero

ex. $E = \begin{bmatrix} 5 & 0 & 0 \\ 4 & 0 & 0 \\ 3 & 2 & 1 \end{bmatrix}$ $\qquad \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$

<u>Upper Triangular</u> Matrix – all entries <u>below</u> diagonal are zero

<u>Zero</u> Matrix – <u>all</u> entries are zero

<u>Vector</u> := a matrix with exactly one column

column vector

ex. $\begin{bmatrix} 1 \\ 2 \\ 9 \\ 1 \end{bmatrix}$ ← each entry is called a <u>component</u>

column vector in $\mathbb{R}^4$

<u>vector space</u> := set of all column vectors with n components denoted by $\mathbb{R}^n$

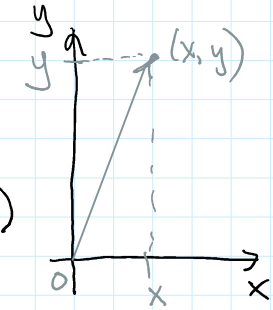
<u>row vector</u> := matrix with exactly one row

ex. $\begin{bmatrix} 1 & 2 & 9 & 9 & 1 \end{bmatrix}$ ← row vector with 5 components

ex. 2×3 matrix $\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$

3 columns

3 vectors in $\mathbb{R}^2$



vectors – standard representation : $\vec{v} = \begin{bmatrix} x \\ y \end{bmatrix}$

directed line segment from origin to (x, y)

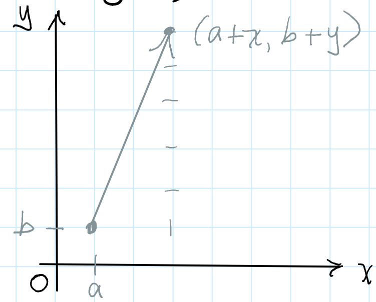
translation of (above) vector :

instead of starting at origin, start line segment at (a, b)

preserve direction

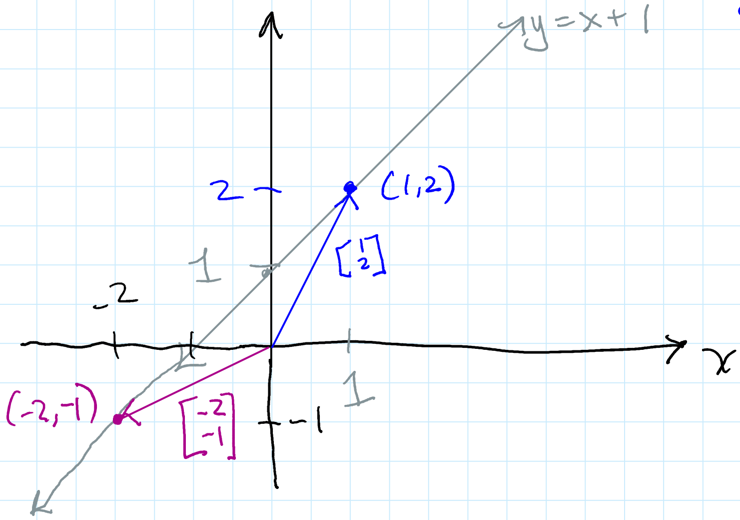
y ↑ $(a+x, b+y)$

instead of starting at origin, start line segment at (a,b)
preserve
- direction
- length



ex. given $\vec{v} = \begin{bmatrix} x \\ x+1 \end{bmatrix}$

then the set of vectors is represented by line $y = x+1$



ex. $(1,2)$ is a solution to $\begin{bmatrix} x \\ x+1 \end{bmatrix}$

ex. when $x=-2 \Rightarrow (-2,-1)$

$\Rightarrow \begin{bmatrix} -2 \\ -1 \end{bmatrix}$

ex. given $\left| \begin{array}{l} x+2y=3 \\ 4x+5y=6 \\ 7x+8y=9 \end{array} \right| \Rightarrow \underbrace{\begin{bmatrix} 1 & 2 \\ 4 & 5 \\ 7 & 8 \end{bmatrix}}_{\text{coefficient matrix}}, \begin{bmatrix} 1 & 2 & \vdots & 3 \\ 4 & 5 & \vdots & 6 \\ 7 & 8 & \vdots & 9 \end{bmatrix}$ or $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$

augmented matrix

ex. solve $\left| \begin{array}{l} x+4y+2z=1 \\ 2x+5y+z=5 \\ 4x+10y-z=1 \end{array} \right| \longrightarrow \begin{array}{c} \begin{array}{cccc} x & y & z & \end{array} \\ \begin{bmatrix} 1 & 4 & 2 & \vdots & 1 \\ 2 & 5 & 1 & \vdots & 5 \\ 4 & 10 & -1 & \vdots & 1 \end{bmatrix} \end{array}$

$\downarrow$

$\begin{bmatrix} 1 & 4 & 2 & 1 \\ 0 & -3 & -3 & 3 \\ 0 & -6 & -9 & -3 \end{bmatrix}$

$\downarrow$

$\begin{bmatrix} 1 & 4 & 2 & 1 \\ 0 & 1 & 1 & -1 \\ 0 & -6 & -9 & -3 \end{bmatrix}$

$\downarrow$

$\begin{bmatrix} 1 & 0 & -2 & 5 \end{bmatrix}$

Do: solve potential solution...

then goal is:

$\left| \begin{array}{l} x = \sim \\ y = \sim \\ z = \sim \end{array} \right|$

$\downarrow$

$\begin{bmatrix} 1 & 0 & 0 & \vdots & \sim \\ 0 & 1 & 0 & \vdots & \sim \\ 0 & 0 & 1 & \vdots & \sim \end{bmatrix}$

$$\begin{bmatrix} & & & \vdots & \\ 0 & 1 & 0 & \vdots & \sim \\ 0 & 0 & 1 & \vdots & \sim \end{bmatrix}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 0 & -2 & 5 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 0 & 0 & 11 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 3 \end{bmatrix} \xrightarrow[\text{as a vector}]{\text{write sol'n}} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ -4 \\ 3 \end{bmatrix}$$

ex. with 5 unknowns

given
$$\left| \begin{aligned} x_1 - x_2 \qquad\qquad + 4x_5 &= 2 \\ x_3 \qquad - x_5 &= 2 \\ x_4 - x_5 &= 3 \end{aligned} \right| \longrightarrow \left| \begin{aligned} x_1 &= 2 + x_2 - 4x_5 \\ x_3 &= 2 \qquad\quad + x_5 \\ x_4 &= 3 \qquad\quad + x_5 \end{aligned} \right|$$

wrote x_1, x_3, x_4 ITO x_2, x_5

let $x_2 = t$

$x_5 = r$

then
$$\begin{aligned} x_1 &= 2 + t - 4r \\ x_3 &= 2 \quad\;\; + r \\ x_4 &= 3 \quad\;\; + r \end{aligned} \longrightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 2 + t - 4r \\ t \\ 2 \quad + r \\ 3 \quad + r \\ r \end{bmatrix}$$